Workshop on branching processes and products of random matrices

LMBA, Université Bretagne Sud, Campus de Tohannic, Vannes, 11-12 juillet 2024

Program

Thursday 11/07/2024

12:00 -14:00 Lunch

14:00-14:30 Reception

14:30 – 15:20 Xinxin Chen (Beijing Normal Univ.)

Title: Precise large deviation of level sets of branching Brownian motion

15:20-16:10 Christophe Cuny (Univ. Brest)

Title: Limit theorems for iid products of positive matrices

16:10- 16:40 Coffee Break

16:40 – 17:30 Hui Xiao (Chinese Academy of Sciences).
Title: Limit theorems for the coefficients of products of random matrices.

19:30 Dinner

Friday 12/07/2024

09:00-09:50 Loic Chaumont (Univ. Angers)

09:50-10:40 Wang Yanqing (Zhongnan Univ. Economics and Law)

10:40-11:10 Coffee break

11:10-12:00 Maxime Ligonniere (Univ. Tours) Kesten-Stigum type results for some multitype Galton-Watson processes in random environment with an infinite number of types.

12:00: Lunch

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Abstract

Talk 1 : by Xinxin Chen (Beijng Normal University), xinxin.chen@bnu.edu.cn
Title: Precise large deviation of level sets of branching Brownian motion

Abstract: We consider a continuous-time branching Brownian motion (BBM) on the real line which spreads with speed $\sqrt{2}$. Its x-level set counts the number of particles located above xt at time t. The typical behavior of x-level set for $0 < x < \sqrt{2}$ is known by Biggins [2] and Glenz et al [3]. We study the precise large deviation of level sets, which improves the previous result of Aïdékon et al[1]. We also discuss the behaviors of BBM conditioned on large x-level set. This is based on joint works with L. de Raphélis and Heng Ma.

References

[1] E. Aïdékon, Yueyun Hu and Zhan Shi (2019). Large deviations for level sets of a branching Brownian motion and Gaussian free fields, *J. Math. Sci.*, New York **238**(4), 348-365.

[2] J. D. Biggins (1992). Uniform Convergence of Martingales in the Branching Random Walk, *Ann. Probab.*, **20**(1), 137-151.

[3] G. Glenz, N. Kistler and M. Schmidt (2018). High points of branching Brownian motion and McKean's Martingale in the Bovier-Hartung extremal process, *ECP*, **23**, 1-12.

Talk 2 : by Christophe Cuny (Univ. Brest), Christophe.Cuny@univ-brest.fr

Title: Limit theorems for iid products of positive matrices

Abstract:

Given iid matrices $(Y_n)_{n\geq 1}$ of size $d \geq 2$, with non-negative entries, we study the asymptotic behaviour of $(\log \|A_n x\|)_{n\geq 1}$ for $x \in (\mathbb{R}^+)^d$ and of $(\log \|A_n\|)_{n\geq 1}$, where $A_n = Y_n \cdots Y_1$ and $\|\cdot\|$ is any norm on $\mathbb{R}^d$. In particular, we focus on the ASIP (almost sure approximation by a brownian motion) with rate and on Berry-Esseen type results. The proofs make use of coupling coefficients introduced in the context of invertible matrices and the results are optimal in terms of moment condition. We also obtain results for the matrix coefficients. Joint work with Jérôme Dedecker and Florence Merlevède.

Talk 3 : by Hui Xiao (Chinese Academy of Sciences, Beijing, China), xiaohui@amss.ac.cn
Title: Limit theorems for the coefficients of products of random matrices.

Abstract: Let $(g_n)_{n \geq 1}$ be a sequence of independent and identically distributed random invertible $d \times d$ matrices and consider the random walk $G_n = g_n \ldots g_1$. The study of the asymptotic behaviors of G_n has attracted much attention since the seminal work of Furstenberg and Kesten (1960), where the law of large numbers and the central limit theorem were established in the special case of positive matrices. In this talk, we will present some recent progress on Berry-Esseen bound, Edgeworth expansion, large and moderate deviations for the coefficients of G_n. We will also give some applications to the study of limit theorems for the first passage time of multivariate perpetuity sequences. Mainly based on joint works with Ion Grama and Quansheng Liu.

Talk 4, by Loic Chaumont (Univ. Angers), loic.chaumont@univ-angers.fr

Title: On the explosion of branching processes indexed by the Esscher transform

Abstract:

Let φ be the branching mechanism of a non conservative continuous state branching process, that is $\int_{0+} d\lambda/|\varphi(\lambda)| < \infty$. It is well known that a continuous state branching process $Z^{(0)}$ with such a branching mechanism is not conservative. In particular its explosion time to ∞ is finite with positive probability. We construct, on the same probability space, a family of conservative continuous state branching processes $Z^{(\varepsilon)}$, $\varepsilon \geq 0$ such that for each ε, $Z^{(\varepsilon)}$ has branching mechanism $\varphi^{(\varepsilon)}(\lambda) = \varphi(\lambda + \varepsilon) - \varphi(\varepsilon)$, and hence converges a.s. to $Z^{(0)}$ as $\varepsilon \to 0$. Then we study the speed of explosion of the family $Z^{(\varepsilon)}$ when $\varepsilon \to 0$. In particular we characterise the functions f with $\lim_{\varepsilon \to 0} f(\varepsilon) = \infty$ and such that the first passage times $\sigma_\varepsilon = \inf\{t : Z_t^{(\varepsilon)} \geq f(\varepsilon)\}$ converge toward the explosion time of $Z^{(0)}$.

This is a joint work with my PhD student Clément Lamoureux.

Talk 5, by Yanqing Wang (Zhongnan University of Ecnomics and Law, Wuhan, China), yanqingwang102@163.com

Title: Limit theorems for a supercritical two-type decomposable branching process in a random environment

Abstract: Let $Z_n=(Z_n^{(1)},Z_n^{(2)})$ be a two-type decomposable branching process in an independent and identically distributed random environment, where a type 1 particle may produce particles of types 1 and 2, while a type 2 particle can only give birth to type 2 particles. We consider asymptotic properties of this process in the supercritical case. Because $Z_n^{(1)}$ is an usual single-type branching process in a random environment, we only consider $Z_n^{(2)}$. First, under some moment conditions, we find a suitable factor Π_n such that the normalized population size $W_n=\frac{Z_n^{(2)}}{\Pi_n}$ converges almost surely to a finite random variable W, and provide a decomposition expression and a non-degeneracy condition of W. Second, we give conditions under which (W_n) is convergent in L^p for $p\geq1$, and bounded in L^p for $0<p<1$. Finally, we establish a central limit theorem for $\log Z_n^{(2)}$.

Talk 6, by Maxime Ligonnière (Univ. Tours), maxime.ligonniere@univ-tours.fr

Titre : Kesten-Stigum type results for some multitype Galton-Watson processes in random environment with an infinite number of types.

Abstract:

Multitytype Galton-Watson processes in random environment (for short, MGWREs) are variants on the well known Galton-Watson process, where the offpsring distribution of an individual of the n-th generation depends both on a notion of *type* assigned to each individual, and on the value of a stochastic process (e_k) at time n, which represents the environment in which the population evolves.

Such process were introduced by Ahtreya and Karlin in 1971. In the case where the *types* of the individuals belong to a finite set X, these process were largely studied, first in the 1970s, and more recently since 2015, following some development in the theory of products of random positive matrices. However, the case of an inifinite number of types remain largely unstudied.

In a previous paper, I proved ergodicity results on a certain class of products of random positive operators on infinite dimensional spaces. These results paves the way for the study of MGWREs with an infinite number of types. In particular, in this talk, I will present a work in progress, focused on some Kesten-Stigum type results. More precisely, in the case where the MGWRE is supercritical, I will describe the population asymptotically on the survival event, both in terms of the growth of the size of the population and of the distribution of the types of the individuals.